

Evolution of Very Massive Stars with Pulsational Mass Loss

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Received January 20, 1970

Evolution with moderate mass loss has been investigated for hydrogen-burning stars whose mass exceeds the critical mass for pulsational stability at the top of the main sequence. A prescription based on minimizing the degree of pulsational instability is used to specify the rate of mass loss. With a zero-age hydrogen abundance of $X = 0.70$ and electron-scattering opacity, stars of initially $60 - 115 M_{\odot}$ become stable before the end of hydrogen burning; more massive stars evolve into unstable pure-helium stars of high mass. The smallest remnant left after hydrogen burning is $12 M_{\odot}$, for an initial mass of $115 M_{\odot}$. The hydrogen-burning tracks stay close to the zero-age main sequence, and the mass-loss rate is remarkably uniform, $3 \times 10^{-5} M_{\odot} \text{ year}^{-1}$. The helium-burning phase is considered for the remnants of initially 100 and $> 115 M_{\odot}$. In contrast with the possible cases of no significant mass loss or very rapid mass loss, the assumption of moderate mass loss seems to yield results which are consistent with the relevant observational data, particularly for the *most luminous* group of Wolf-Rayet stars.

Key words: stellar evolution — pulsation — mass loss — Wolf-Rayet stars

I. Introduction

Zero-age main-sequence stars with masses exceeding a certain critical mass become unstable against nuclear-energized pulsations. It is well known that such stars, evolving without mass loss, will be stabilized well before the end of their hydrogen-burning phase. On the other hand, pulsational rise times on the main sequence are short enough that the possibility arises of pulsationally driven loss of mass before the instability can be nullified. If this is indeed the case, then the time over which the instability persists might be substantially increased, as indicated by the following argument.

During the time that matter is being ejected from the stellar surface, interior evolution proceeds normally and the core shrinks, leaving behind a region of inhomogeneous composition. The consequent increase in central condensation has a stabilizing tendency. On the other hand, the inhomogeneous region contains a helium-rich composition which comprises an increasing fraction of the envelope material as zero-age composition is lost from the surface. The result is a gradual replacement of hydrogen by helium as the main chemical constituent of the envelope. This increase in mean molecular weight has a destabilizing tendency. Thus, provided that it occurs at a favorable rate, the mass loss itself could provide a means whereby the instability is maintained.

Whether this actually occurs depends, of course, on the rate of ejection of matter. If this rate is either too high or too low, the lifetime of the phase of pulsational instability will be minimal, as illustrated by the following two extreme cases.

No Mass Loss (Case A). The pulsational instability never reaches a violence sufficient for disruption of the star. This will be the case if higher-order effects limit the instability to a standing wave of finite amplitude. In that event, the star remains intact, and stability ensues as soon as the central condensation attains a large enough value.

Extremely Rapid Mass Loss (Case B). The pulsation grows rapidly and severe mass loss occurs. The stellar mass drops below the critical limit before an appreciable amount of hydrogen has been burned in the core, and the instability disappears.

The evolutionary tracks for stars at these two extremes are well known. In the former case, the track is simply that of a normally evolving star at constant mass, while, in the latter case, evolution proceeds directly down the main sequence to a point corresponding to the critical mass, and subsequently along the normal track for a star of that mass.

It is the purpose of the present work to calculate the evolutionary tracks of stars for rates of mass loss intermediate between the two extremes noted

above and to compare these tracks with the relevant observations. Such an intermediate case of mass loss will be defined as follows.

Moderate Mass Loss (Case C). The rate of mass loss is such that the lifetime of the phase of pulsational instability approaches (or exceeds) the whole hydrogen-burning lifetime.

II. Prescription for Mass Loss

The critical mass for pulsational instability on the zero-age main sequence is given by (Schwarzschild and Härm, 1959)

$$M_{\text{crit}}^{(1)}/M_{\odot} = 21 \mu_e^{-2} \quad (1)$$

where μ is the mean molecular weight and the opacity source is due only to electron scattering. A star will remain pulsationally unstable if its mass stays above the critical mass for each stage of evolution (the stage being specified by the central hydrogen abundance X_c). A *lower limit* to the critical mass for any stage X_c is equal to the critical mass for all chemically homogeneous stars with $X = X_c$, as given by Eq. (1). An *upper limit* is equal to the critical mass for all inhomogeneous stars which reach stage X_c under evolution at constant mass, as given by the following fitted formula (valid in the range $\mu_c/\mu_e = 1.0 - 1.5$):

$$M_{\text{crit}}^{(2)}/M_{\odot} = 21 \mu_e^{-2} (2.76 - 1.76 \mu_c/\mu_e)^{-1/4}. \quad (2)$$

For example, the critical mass for stars on the normal zero-age main sequence occurs at $\sim 60 M_{\odot}$. But the critical mass for stars at a stage when their central hydrogen content is half depleted may occur anywhere between 30 and $240 M_{\odot}$, depending on how much hydrogen remains in the envelope. Thus, while a star of $100 M_{\odot}$ is violently unstable on the zero-age main sequence, the star may or may not be unstable at a later stage of evolution, depending on the degree of mass loss.

Since significant mass loss from a pulsationally unstable star will certainly involve finite-amplitude pulsations, the mass-loss rate cannot be determined in a linear pulsation theory. Because of this, one must either treat directly the necessary hydrodynamics or else make some more or less arbitrary assumptions concerning the amount of matter ejected. The most arbitrary assumption possible involves a simple specification of the rate of mass loss. Using such a specification, a number of authors have calculated evolutionary tracks of massive

stars ejecting matter (Tanaka, 1966a, b; Hartwick, 1967).

A less arbitrary approach was adopted by Osaki (1966) in a study of supermassive stars. He used the gain of pulsational energy per cycle, as calculated in a linear theory, to estimate the rate of mass loss. The rate thus obtained was very high, yielding a result close to that of Case B described above.

The mass loss prescription that we shall adopt is designed to ensure that the lifetime of the phase of pulsational instability is maximal. We shall do this by *assuming* that the star keeps itself as long as possible in a state of marginal instability by losing mass. To that end let us specify that, at each step, a given amount of matter will be lost on the *longest possible time scale* consistent with continuing instability. For convenience, we take the amount of mass loss to be all the matter outside the convective core of a given unstable model. This fixes the *actual* mass of the subsequent model. On the other hand, the *critical* mass for this model will depend on how far along the evolutionary sequence we choose it to be, i. e., on the time step selected for the mass loss. Since internal evolution is a stabilizing factor, the upper limit for this time step will always be determined by the requirement that the critical mass of the resulting model never be permitted to exceed its actual mass. The quantitative criteria for this prescription will be described in Section III.

Such a scheme, although arbitrary, is self-consistent insofar as the continuing instability provides at each step a possible mechanism to account for the mass ejection. As we shall see, the rate of mass loss which emerges is remarkably constant and has a physically plausible value; further, evolutionary tracks of stars evolving according to this scheme are very different from the tracks resulting from either Case A or Case B.

III. Model Sequences

Models for the hydrogen-burning evolution of very massive stars, relevant to Cases A and B, have already been constructed by Blackler (1958), Schwarzschild and Härm (1958), Stothers (1966b), and Ezer and Cameron (1967).

Case C is considered here. According to our adopted prescription for mass loss, three model sequences have been constructed, each beginning with a pulsationally unstable main-sequence star of zero-age composition $X_e = 0.70$, $Z_e = 0.03$, $X_{\text{CNO}} = Z_e/2$. The initial masses chosen were 100,

130 and 200 M_{\odot} . The models were constructed according to the method described in Stothers (1963), except that semiconvection was ignored and the region in which it occurred was treated as radiative. The equation of state was taken as the sum of perfect-gas pressure and radiation pressure; the opacity was assumed due only to electron scattering. The latter assumption results in an underestimation of the critical masses for each stage of evolution (Stothers and Simon, 1970). However, in view of the uncertainty of the mass loss rate and initial chemical composition, this is unimportant. The nuclear parameters for the CNO bi-cycle of energy generation were those adopted by Stothers (1966b).

Radial pulsation characteristics have been calculated in the usual linear quasi-adiabatic approximation (e.g. Schwarzschild and Härm, 1959). The degree of stability (or instability) of the fundamental mode is evaluated in terms of the net gain or loss rate of pulsational energy per cycle L_p^*/L , the asterisk indicating our neglect of running waves at the surface; this neglect is not important (Schwarzschild and Härm, 1959; Simon and Stothers, 1969a). The pulsational e -folding time $1/K^*$ follows from L_p^* and the dimensionless eigenfrequency ω^2 . Notation and assumptions are the same as in Simon and Stothers (1969a).

In any evolutionary sequence, a given model was always determined by assigning it a mass equal to the mass of the convective core of the previous model and specifying a central hydrogen abundance X_c . The run of chemical composition in the new envelope is described by

$$\mu = \mu_R q^{-\lambda}, \quad (3)$$

where q stands for mass fraction and the subscript R indicates the surface. Since each model gives rise to the next by ejecting its entire envelope, a single value of λ , per model, suffices to describe the composition gradient and must be solved for as an eigenvalue.

Each trial model thus constructed was examined for pulsational stability: if it was found not to be "marginally unstable", it was rejected and another model with a different X_c was calculated¹⁾. A model

¹⁾ For trial models it was not necessary to solve the pulsation equations in order to estimate the degree of stability. Instead, use was made of a general quasi-linear relationship which exists between L_p^*/L and a well known upper limit for the fundamental eigenfrequency of radial pulsation, which may be evaluated from the equilibrium model alone (Simon and Stothers, 1969b).

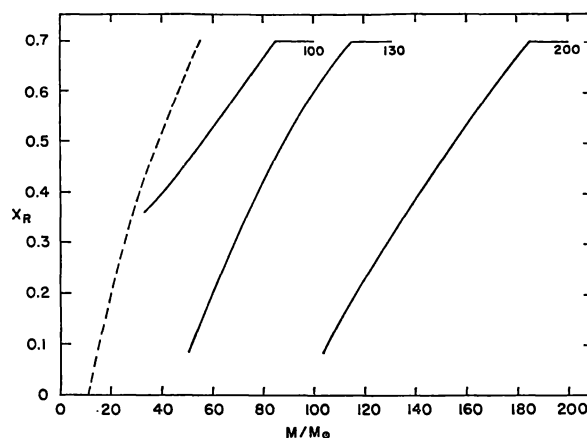


Fig. 1. Evolution of marginally unstable models (Case C) during core hydrogen depletion, on a plot stellar mass vs. surface hydrogen abundance. Dashed line represents chemically homogeneous models ($Z = 0.03$)

was considered acceptable if L_p^*/L lay in the range 0.0 to 1.0. Because of the rapid change of L_p^*/L with small changes in the model structure, the structure can be determined to within rather narrow bounds. A sequence of mass-loss models was terminated when (1) no unstable configuration could be found or (2) core hydrogen became essentially exhausted.

The elapsed time between models was calculated according to the expression

$$\Delta\tau = \int_{X_c}^{X_c^i} (E M_f/L) dX_c, \quad (4)$$

where the superscript i indicates the previous model, the subscript f the boundary of the convective core, and $E = 6.0 \times 10^{18}$ ergs/gm. The model sequences for Case C are presented in Tables 1 and 2, while a comparison of lifetimes for Cases A, B, and C is made in Table 3.

a) Minimum Initial Mass of Stars Which Do Not Become Stable During Evolution

The evolutionary sequences for stars of initially 130 and 200 M_{\odot} reach the stage of core hydrogen exhaustion in unstable configurations, while the sequence for a star of initially 100 M_{\odot} is stabilized when the central hydrogen content drops to half its original value. Since the critical mass for a homogeneous hydrogen-burning star with $X \sim 0$ is 12 M_{\odot} (Eq. 1), it is clear that both the 130 and 200 M_{\odot}

Table 1. *Models for 100 M_{\odot} with mass loss during core hydrogen-burning (Case C)*

	Initially 100 M_{\odot}							
	0	1	2	3	4	5	6	7
M/M_{\odot}	100.0	85.0	67.2	53.3	41.9	32.6	32.6	32.6
M_1/M_{\odot}	84.6	67.2	53.3	41.9	32.6	24.6	20.9	18.7
λ	...	0.432	0.360	0.308	0.187	0.115	1.179	1.369
X_R	0.700	0.700	0.577	0.483	0.407	0.361	0.361	0.361
X_c	0.700	0.577	0.483	0.407	0.361	0.330	0.170	0.062
β_c	0.523	0.518	0.531	0.547	0.576	0.614	0.547	0.498
$\log T_c$	7.615	7.619	7.621	7.622	7.619	7.614	7.639	7.671
$\log \varrho_c$	0.161	0.208	0.272	0.336	0.399	0.464	0.504	0.580
$\varrho_c/\langle\varrho\rangle$	22.5	29.6	27.8	26.7	24.4	23.0	41.8	73.6
$\log (L/L_{\odot})$	6.141	6.090	6.000	5.900	5.770	5.610	5.700	5.750
$\log (R/R_{\odot})$	1.113	1.114	1.049	0.989	0.920	0.853	0.927	0.984
$\log T_e$	4.742	4.729	4.739	4.744	4.746	4.739	4.725	4.709
τ (10^6 yr)	0.00	0.69	1.18	1.58	1.82	1.99	2.89	3.35
ω^2	2.309	2.854	2.811	2.843	2.844	2.988
Period (hr)	8.57	8.36	7.59	6.88	6.11	5.37
L_p^*/L	3.61	0.45	0.78	0.70	0.90	0.35
$1/K^*$ (yr)	1500	7300	4000	4200	3500	9700

Table 2. *Models for 130 M_{\odot} and 200 M_{\odot} with mass loss during core hydrogen-burning (Case C)*

	Initially 130 M_{\odot}					Initially 200 M_{\odot}			
	0	1	2	3	4	0	1	2	3
M/M_{\odot}	130.0	115.0	93.2	75.8	62.0	200.0	185.0	151.0	125.0
M_1/M_{\odot}	115.0	93.2	75.8	62.0	51.2	185.0	151.0	125.0	104.0
λ	...	0.638	0.737	0.853	0.989	...	0.984	1.103	1.285
X_R	0.700	0.700	0.541	0.381	0.227	0.700	0.700	0.467	0.268
X_c	0.700	0.541	0.381	0.227	0.086	0.700	0.467	0.268	0.085
β_c	0.474	0.452	0.435	0.412	0.384	0.400	0.353	0.322	0.285
$\log T_c$	7.624	7.632	7.643	7.659	7.687	7.637	7.652	7.670	7.704
$\log \varrho_c$	0.103	0.146	0.215	0.296	0.410	0.012	0.056	0.140	0.268
$\varrho_c/\langle\varrho\rangle$	23.5	34.1	36.9	40.0	43.2	25.5	47.9	51.0	60.1
$\log (L/L_{\odot})$	6.320	6.310	6.285	6.270	6.265	6.598	6.630	6.635	6.650
$\log (R/R_{\odot})$	1.177	1.198	1.157	1.112	1.056	1.281	1.347	1.298	1.252
$\log T_e$	4.755	4.742	4.756	4.775	4.802	4.772	4.748	4.773	4.800
τ (10^6 yr)	0.00	0.78	1.44	1.98	2.40	0.00	0.92	1.54	2.00
ω^2	2.082	2.675	2.714	2.704	2.640	1.772	2.584	2.448	2.422
Period (hr)	9.85	9.97	9.51	9.03	8.33	12.4	13.3	12.8	12.1
L_p^*/L	4.40	0.92	0.61	0.50	0.62	5.67	0.65	1.08	0.92
$1/K^*$ (yr)	1100	2800	3000	2600	1500	790	2600	1100	830

sequences, which end their core hydrogen-burning phase with homogeneous masses of about 50 and 100 M_{\odot} , respectively, have continually had a large possible margin for instability. Obviously, if a sequence is to become stabilized precisely at the point of core hydrogen exhaustion, the initial mass must lie somewhere between 100 and 130 M_{\odot} . We shall now evaluate the precise initial mass.

In Fig. 1, the evolution of our calculated sequences is shown on a plot of surface hydrogen abundance against stellar mass. It is obvious from the figure that the slope $dX_R/d(M/M_{\odot})$ determined from the post-zero-age models is approximately the same for all three sequences. Adopting the mean slope of $dX_R/d(M/M_{\odot}) = 0.008$, we find that a star which ends its evolution at 12 M_{\odot} with $X_R = 0$

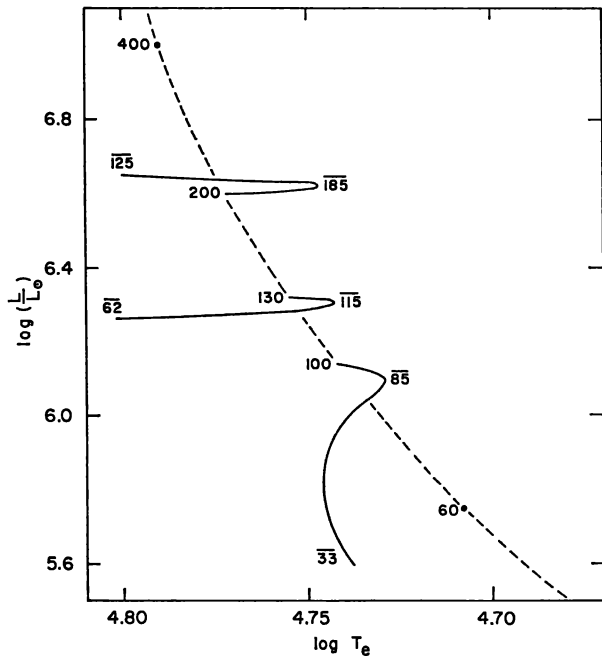


Fig. 2. Theoretical H-R diagram for marginally unstable models (Case C) during core hydrogen depletion. Masses are indicated in solar units. Dashed line represents the zero-age main sequence

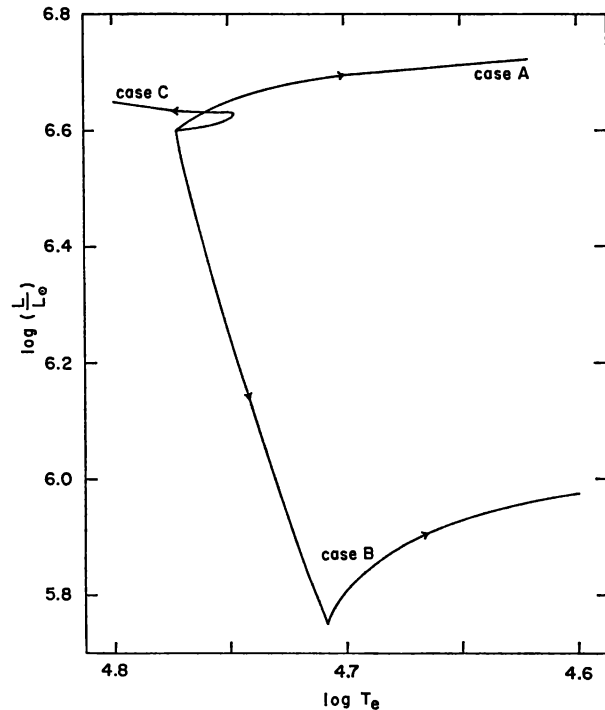


Fig. 3. Theoretical H-R diagram for $200 M_{\odot}$ (illustrating the three possible cases of mass-loss) during core hydrogen depletion

Table 3. *Evolutionary lifetime of core hydrogen-burning* (10^6 yr)

M/M_{\odot}	Case A	Case B	Case C
60	3.1	3.1	3.1
100	2.5	3.1	3.4
130	2.3	3.1	2.5
200	2.0	3.1	2.1

has an initial *core* mass of $100 M_{\odot}$. The *envelope* mass is derived from the simple fact that homogeneous stars of high mass have nearly the same mass contained in their envelopes,

$$(1 - q_r) M/M_{\odot} = 6.0 \mu_e^{-2}, \quad (5)$$

approximately valid in the range $(M/M_{\odot}) \mu_e^2 = 21 - 150$ but with a gradual maximum around $(M/M_{\odot}) \mu_e^2 = 60$ (Stothers, 1966b). For our chosen zero-age chemical composition, this envelope mass is $15 M_{\odot}$. Hence the initial mass of a star which becomes stabilized precisely at the point of core hydrogen exhaustion must be $115 M_{\odot}$.

b) Evolutionary Tracks for Marginal Instability

Evolutionary tracks of our models are shown in a series of three H - R diagrams. Figure 2 shows the core hydrogen-burning phase of evolution for all three masses. The effective temperature initially drops when the zero-age envelope is removed. Subsequently, the temperature changes very slowly because of the competition between the decreasing mass, which tends to lower the temperature, and the increasing mean molecular weight, which tends to raise it. The luminosity is affected similarly by the same competition, but, whereas the mean molecular weight has a greater influence on the effective temperature, the mass has a greater influence on the luminosity (for initial masses less than $\sim 160 M_{\odot}$). Above $160 M_{\odot}$, the effect of mean molecular weight dominates both the luminosity and effective temperature since the percentage of the original mass which is lost during evolution is less at higher masses. (The maximum percentage of mass loss actually occurs at $115 M_{\odot}$ and amounts to 90 per cent.) However, the nearly compensating effects of mass loss and helium enrichment result

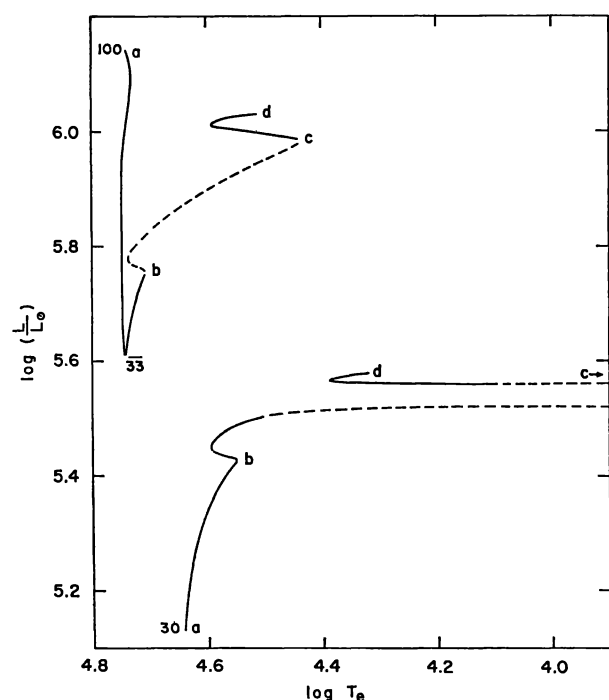


Fig. 4. Theoretical H-R diagram for $100 M_{\odot}$ (Case C) and $30 M_{\odot}$ during the following phases: $a-b$, core hydrogen depletion; $b-c$, core hydrogen exhaustion and core contraction; $c-d$, core helium depletion. Sequence starting at $100 M_{\odot}$ is stabilized when the mass reaches $33 M_{\odot}$. Dashed segments are schematic only

in little *observable* change on the H - R diagram, in contrast to the changes which are obtained at lower initial masses (15 and $47 M_{\odot}$) for rather different rates of mass loss (Tanaka, 1966a, b; Hartwick, 1967).

c) Evolution of $200 M_{\odot}$

Tracks on the H - R diagram of the star with initially $200 M_{\odot}$ are shown for the three possible cases of mass loss in Fig. 3. Case A (no mass loss) results in the standard track as calculated elsewhere (Stothers, 1966b). Case B (fast mass loss) is illustrated by evolution directly down the initial main sequence to $60 M_{\odot}$ and then along the normal path (Stothers, 1966b). Case C corresponds to the intermediate case which we have calculated, where evolution proceeds in a marginally unstable configuration, ending in a pure-helium star of $100 M_{\odot}$.

Subsequently, the star must contract to the "helium main sequence." At that point the star will become once again violently unstable, the

critical mass for a pure-helium star with electron-scattering opacity being only $7-9 M_{\odot}$ (Boury and Ledoux, 1965; Noels-Grötsch, 1967). Independently of the rate of mass loss, the object must evolve approximately down the helium main sequence since the computed tracks of helium stars evolving at constant mass never stray far from the original sequence (Deinzer and Salpeter, 1964). If the star continues to lose mass during helium depletion, the carbon star which results might also become unstable when carbon burning commences (Noels-Grötsch *et al.*, 1967).

d) Evolution of $100 M_{\odot}$

The evolutionary sequence for a mass of initially $100 M_{\odot}$ is stabilized, in Case C, before the end of core hydrogen burning, when the mass has fallen to $33 M_{\odot}$. The further evolution has been followed by taking the mass to be constant (Fig. 4). It is of interest to note that the initial stage of the stable phase is characterized by a nearly homogeneous distribution of chemical composition with $X = 0.33$. The evolution therefore proceeds qualitatively as for a normal main-sequence star. However, the lifetime is only a third of the lifetime of a normal star having the same mass (Stothers, 1963) because of the higher luminosity and smaller initial hydrogen content in the present case.

We have also calculated the later phase of helium burning in the core, in order to compare our results with the results for a normal star of similar mass (Stothers, 1966a). The luminosity and lifetime of core helium burning depend on the *core mass*, which is here approximately that of a normal star of $60 M_{\odot}$ (Stothers and Chin, 1968). The present core mass eventually encompasses 82 per cent of the total mass and burns for 3.8×10^5 yr. On the H - R diagram evolution proceeds in the usual fashion, reaching a maximum effective temperature when $Y_c = 0.3$. However, this temperature is higher than for any normally evolving star because of the hydrogen-poor character of the envelope. Only one qualitative difference exists between the sequence presented here and the sequences obtained for normal stars: the hydrogen-poor star evolves directly into the blue-supergiant phase of helium burning without first becoming a red supergiant. We have verified this point by recalculating the initial helium-burning model with the proper inclusion of bound-free absorption in the opacity (see Stothers and Chin, 1968).

IV. Comparison with Observations

Various mechanisms of mass ejection have been proposed to account for the peculiar properties of the Wolf-Rayet (W – R) stars. These mechanisms have included: a large steady stellar wind (Tanaka, 1966a, b), equatorial mass loss due to fast rotation (Limber, 1964), and envelope stripping due to interaction of the components in a close binary system (Paczynski, 1967). In the present paper, we propose a fourth mechanism: pulsational mass loss. While an earlier application of this idea to P Cygni (Schwarzschild and Härm, 1959) seems now to be unjustified (Stothers and Simon, 1968), we might nevertheless expect to find such pulsationally destabilized objects among the *most luminous* W – R stars.

Classical W – R stars are found to occupy a broad range of luminosity, but two major luminosity groups may nevertheless be distinguished (Smith, 1968b). Those having the latest spectral subdivisions compose the bright group, lying in the interval $M_v = -6$ to -8 (Smith, 1968a), and are of primary interest here. The faint group will not be discussed here, but may arise from the binary mechanism of mass exchange (Paczynski, 1967).

The binary mechanism, however, runs into a number of difficulties accounting for the *most luminous* objects. These difficulties are as follows. First, if a star of the highest stable mass on the main sequence ($60 M_\odot$) evolves and loses most of its envelope to a companion, then its final mass will be $\sim 25 M_\odot$. The visual absolute magnitude of the remnant is $M_v = -5.5$ for $T_e = 50000^\circ\text{K}$, the effective temperature being a general result for various binary mass-exchange models (Smith, 1968b) as well as that inferred from actual observations of luminous W – R stars (Kuhi, 1968). The predicted limit of $M_v = -5.5$ is compatible with the low-luminosity group of W – R stars but seemingly fails to account for the high-luminosity group. Second, if pulsation of the evolved remnant is responsible for the W – R phenomenon, then the coolest expected effective temperature is far too hot, 110000°K , a consequence of the small radius required to avoid pulsational damping (Simon and Stothers, 1969a). Third, absolute magnitudes as bright as $M_v = -10$ are known for composite objects (OB + WN) in the Large Magellanic Cloud (Smith, 1968a). Both the high luminosity and the luminosity-classification criteria for the OB star require it in several cases to be a supergiant, hence

more evolved than the W – R star. Fourth, there is evidence for the existence of *some* single W – R stars, not only by analogy with the W – R nuclei of planetary nebulae but also by virtue of the large emission-line widths of apparently single objects compared with the emission-line widths of known binaries (Hiltner and Schild, 1966; Kuhi, 1968).

The pulsational theory presented here (Case C) eliminates the foregoing difficulties with the brightest W – R stars and accounts for many other associated W – R phenomena if very high masses for some single (or double) W – R stars are admitted. The fact that a number of the most luminous objects are known to be binaries is irrelevant in the context of the present theory because stars of high mass are normally found in binary systems (so that our theory could, in principle, account for any mass ratio $M_{\text{WR}}/M_{\text{OB}} > 12/115 = 0.1$). Although the results of our models should not be taken literally (because of the considerable theoretical uncertainties), they should nevertheless give an idea of the trend of evolution for slow pulsational mass loss.

Masses for W – R stars are poorly known. The members of binary systems seem to have a mass approximately one-third the mass of their OB companions, but even this is uncertain (Smith, 1967a). Since the W – R stars with measured masses are rather faint, the derived average mass of $5 - 10 M_\odot$ (Smith, 1968b) should not be taken as representative of the *most luminous* W – R stars. Blaauw (1961) has indirect evidence from the “runaway” OB stars of the possibility of masses up to $1000 M_\odot$ for the hypothesized former companions of the runaway stars.

The effective temperature and surface gravity of bright W – R stars seem to be very characteristic of main-sequence stars of high mass. Kuhi (1968) gives $T_e = 50000^\circ\text{K}$ and $\log g = 4.0$, to be compared with $T_e = 55000^\circ\text{K}$ and $\log g = 4.3$ from our models. Radii are more uncertain, lying in the range $10 - 20 R_\odot$, but agree well with our models.

According to an unpublished investigation of the composite luminosity function for normal O stars in young clusters, a cutoff occurs at high luminosity which is faster than the gradual tapering expected by fitting the bulk of the O stars to an exponential representation of the luminosity function; a simple extrapolation of the fitted luminosity function predicts 2 “superluminous” objects per 20 normal O-type stars. If these are identified with the most luminous W – R stars, it is then relevant to note that observations of the I Sco association show, in

fact, 2 very bright W – R stars among 29 O-type stars (Schild, 1968). Available data from the youngest associations of the Large Magellanic Cloud also support a ratio of this general order (Westerlund, 1961, 1964). Stellar statistics in the solar neighborhood show 2 intrinsically very luminous W – R stars (Underhill, 1966) within a volume containing about 30 O-type stars (assuming $M_v = -7$ and $A_v = 1$ mag/kpc for the W – R stars). Thus, the most luminous W – R stars may form an extension of the main sequence earlier than spectral type O5.

The cutoff in absolute magnitude for “single” W – R stars occurs at $M_v = -8$. This implies extraordinarily high masses – much higher than those expected to be observed for Case B. For example, theoretical models of $60 M_\odot$ evolve from $M_v = -5.5$ to -6.5 during core hydrogen burning on the main sequence, but $400 M_\odot$ and $200 M_\odot$ (evolving at constant mass) are required to attain $M_v = -8$ at the onset and termination of core hydrogen burning, respectively. Starting with “normal” masses, binary mass exchange can, at most, create masses up to $95 M_\odot$. Thus we conclude that the existence of main-sequence stars with very high masses must be a *genetic* effect. If this is so, then we are obliged to account for the absence of blue supergiants with masses in excess of $\sim 60 M_\odot$ (Stothers and Simon, 1968). While this is difficult to explain by Case A and obvious for Case B, such an absence of very massive (helium-burning) supergiants is predicted as a consequence of our Case C, because models with initial masses $< 115 M_\odot$ become supergiants with luminosities and masses less than those of a normal $60 M_\odot$ supergiant and more massive models (initially) end up as visually faint, pure-helium subdwarfs to the *left* of the main sequence.

The extreme youth of the most luminous W – R stars is attested by much additional observational evidence (see Smith, 1968b). Their location at the tip of the main-sequence turnoff in extremely young clusters is compatible with the ages we have theoretically derived, $2 - 3 \times 10^4$ yr. Even the presence of *evolved* companions (supergiants), where, according to our theory, most of these companions should have initial masses less than the initial mass of the W – R stars, is accounted for by our result that hydrogen-burning times for stars of initially super-critical mass can, in some cases, be *longer* than hydrogen-burning times for stars of normal mass (Case C of Table 3).

The WN and WC spectral sequences, as well as the strength of helium lines, are accountable in our theory as abundance effects. During the earliest stages of evolution while the initial envelope is being *rapidly* lost, the surface composition is simply the zero-age one ($N/C \sim 0.3$). But during the longer, later stages of core hydrogen depletion, the surface composition becomes progressively more nitrogen-enriched as regions are exposed which were formerly in the convective core and thus participating in the CNO bi-cycle reactions. Carbon (and oxygen) re-enrichment will not occur unless the star evolves into a pure-helium star and continues to lose mass, eventually exposing a core in which the helium-burning reactions have taken place (cf. Paczynski, 1967); however, the helium-burning lifetime is short compared to hydrogen burning. We therefore expect that the ratio of numbers of WN and WC stars among the brightest W – R stars should be rather large. Our models suggest a ratio of 3:1 while various observational surveys indicate a ratio between 4:1 and 7:1 (Roberts, 1958; Westerlund and Smith, 1964). In view of the uncertainties, the difference is not considered significant.

The periods associated with the pulsational instability of our models lie in the range 6 – 12 hours. Irregular spectral line variations on a time scale of hours are known in many W – R stars (Smith, 1955; Kuhl, 1968) but continuum variations, if any, in the visual part of the spectrum are apparently less than 0.01 mag (Smith, 1968b). It is, however, in the ultraviolet continuum that any significant manifestation of underlying pulsations is expected, as demonstrated in the Appendix. Radial-velocity variations due to the pulsations themselves will be hard to detect if the star is “buried” in a nebulous envelope.

The theoretical rate of mass loss expected from our models may be computed from the data in Tables 1 and 2 and is

$$-dM/dt \sim 3 \times 10^{-5} M_\odot \text{ yr}^{-1}. \quad (6)$$

This is surprisingly constant for all our models and may be considered acceptable on energetic grounds since

$$-\left(\frac{GM}{R} + \frac{1}{2}v_\infty^2\right) \frac{dM}{dt} < L_P^* \quad (7)$$

if $L_P^* \sim L$ and $v_\infty < 10^4$ km/s. Underhill (1969) has recently estimated observationally a typical loss of $3.5 \times 10^{-5} M_\odot \text{ yr}^{-1}$ from the rapidly expanding envelope surrounding W – R stars (which

show $v_\infty \sim 10^3$ km/s), but she regards this estimate as good only to an order of magnitude.

Additional evidence for mass loss from W – R stars comes from the direct observation of ring nebulae around several of them (unfortunately mostly in the low-luminosity group). The masses of the nebulae range from 7 to $700 M_\odot$ (Johnson and Hogg, 1965; Smith, 1967b). Since they have the form of very thin shells, much of the mass is probably due to interstellar matter swept up by the ejecta from the W – R star. It seems that sufficient material may be ejected or swept up to form condensations, resulting in vast “stellar rings” around some W – R stars (Schmidt-Kaler, 1968). However, one W – R nebula in a clear region of the Small Magellanic Cloud is apparently due only to stellar emission and has $12 M_\odot$ (Westerlund and Henize, 1963). Another massive nebula shows remarkably large changes in radial velocity, probably attributable to the central star (Westerlund, 1960). The foregoing evidence suggests continual violent ejection of matter under abnormal circumstances, since the ordinary O stars do not show nebulae or expanding shells.

Combining our proposal here with our earlier work on the nature of the W – R stars, we envisage the following set of circumstances. Many of the W – R stars in the high-luminosity group could be pulsationally unstable stars of extremely high mass on the main sequence burning hydrogen in their cores, as proposed here. A few W – R stars in the high-luminosity and low-luminosity groups may be massive stars burning helium in their cores and possessing sufficiently thin hydrogen-poor envelopes to be pulsationally unstable (Simon and Stothers, 1969a); however, these objects, near the “helium main sequence,” would show extremely hot surfaces and cannot, therefore, account for *most* of the W – R stars. But such objects could at least arise, in some cases, as a result of mass ejection from the supermassive stars considered in this paper and, in other cases, as a result of mass exchange in massive close-binary systems as considered by Paczynski (1967). It appears unlikely that a radial pulsation mechanism can account for those W – R stars in the low-luminosity group which have moderate surface temperatures. For these stars, other sources of instability must be sought.

Acknowledgements. One of us (N.R.S.) gratefully acknowledges the support of a NAS-NRC Research Associateship under the National Aeronautics and Space Administration.

S. G. Ungar is thanked for compiling the data necessary to extrapolate the main-sequence luminosity function.

Appendix The Observed Pulsation Amplitudes

Possible boundary conditions for the pulsational amplitudes at the surface have been discussed by Baker and Kippenhahn (1965). For simplicity, we shall adopt as the mechanical boundary condition

$$\delta P/P = -(4 + \omega^2) \delta R/R, \quad (\text{A-1})$$

where $\omega^2 = (2\pi/\text{Period})^2 R^3/GM$, and as the thermodynamical boundary condition a linearization of the Planck function. The latter linearization will permit us to evaluate the intensity amplitude as a function of wavelength and temperature; in other words, to appreciate the effect of bolometric correction. Lucy (1966) appears also to have studied this effect, but published no details.

We start with the radial-velocity amplitude, which is, at any point on the stellar surface,

$$2 |v_{\max}| = (4 \pi R/\text{Period}) \delta R/R. \quad (\text{A-2})$$

The *observed* radial-velocity amplitude, integrated over the stellar disk, is smaller than the local radial-velocity amplitude normal to the surface, by a factor 17/24 (Rosseland, 1949). Thus,

$$2K = (17/24) (4 \pi R/\text{Period}) \delta R/R. \quad (\text{A-3})$$

Next, we turn to the intensity amplitude, which will be obtained by linearizing the Planck function, $B_\lambda(T)$, in

$$L_\lambda = 4 \pi R^2 B_\lambda(T). \quad (\text{A-4})$$

For convenience, let the temperature dependence of the Planck function be represented as

$$B_\lambda(T) = C_\lambda T^s, \quad \text{where} \quad s = \frac{1.44/\lambda T}{1 - \exp(-1.44/\lambda T)} \quad (\text{A-5})$$

and C_λ is a function of wavelength only. For $(1.44/\lambda T) \ll 1$, $s = 1$ (Rayleigh-Jeans limit), whereas for $(1.44/\lambda T) \gg 1$, $s = 1.44/\lambda T$ (Wien limit). The Stefan-Boltzmann law for integrated light gives $s = 4$. Perturbing Eq. (A-4), we find

$$\frac{\delta L_\lambda}{L_\lambda} = 2 \frac{\delta R}{R} + s \frac{\delta T}{T}. \quad (\text{A-6})$$

By virtue of Eq. (A-1) and the assumed relation $\delta T/T = (1 - 1/\Gamma_{\text{eff}}) \delta P/P$, we obtain for the total

magnitude variation

$$\Delta m_{\lambda} = -5 (\log e) \frac{\delta L_{\lambda}}{L_{\lambda}} = 5 (\log e) \cdot \left[s (4 + \omega^2) \left(\frac{\Gamma_{\text{eff}} - 1}{\Gamma_{\text{eff}}} \right) - 2 \right] \frac{\delta R}{R}. \quad (\text{A-7})$$

Division of Eq. (A-3) into Eq. (A-7) yields

$$\frac{\Delta m}{2K \text{ (km/s)}} = 0.00126 \cdot \left[s (4 + \omega^2) \left(\frac{\Gamma_{\text{eff}} - 1}{\Gamma_{\text{eff}}} \right) - 2 \right] \frac{\text{Period (hr)}}{R/R_{\odot}}. \quad (\text{A-8})$$

No apparent magnitude variation will occur if s is equal to

$$s_0 = \frac{2}{4 + \omega^2} \left(\frac{\Gamma_{\text{eff}}}{\Gamma_{\text{eff}} - 1} \right). \quad (\text{A-9})$$

In the case of adiabatic pulsations, $\Gamma_{\text{eff}} = \Gamma_2$ where Γ_2 is the second generalized adiabatic exponent. Although pulsations of the surface layers of stars are highly non-adiabatic, the emergent flux variation is actually determined at a slightly deeper (and more nearly adiabatic) layer of hotter temperature, T_a (Castor, 1968; Stothers and Simon, 1970).

In massive blue stars, hydrogen and helium are completely ionized out to the surface, and so $\Gamma_2 = 4/3$ to $5/3$ in all layers. Our "critical" models are characterized by $\omega^2 \sim 3$, hence $s_0 = 8/7$ to $5/7$ if $\Gamma_{\text{eff}} = \Gamma_2$. The lower limit for s_0 is not a physically possible value of s , but the upper limit is if $(1.44/\lambda T) \ll 1$. In the limiting case of maximum pulsational instability ($\omega^2 = 0$), one finds $s_0 = 2$ to $5/4$, which are both physically possible values of s . For the very massive stars considered in this paper, $T_a > T_e = 55000^\circ\text{K}$, which, with the adoption of $\lambda = 4400 \text{ \AA}$ (B magnitude), yields $s < 1.3$ — a value close to s_0 . Therefore it is quite possible that a sufficiently unstable star with high radiation pressure and high surface temperature will exhibit no magnitude variation, or may even exhibit a *negative* correlation of radial-velocity amplitude with light amplitude, if the observations are made at the usual wavelengths (e.g. the effective wavelengths of the U , B , V system).

The smallness of the predicted $\Delta m_{\lambda}/2K$ is one basis for our belief that radial pulsations should not be ruled out as a possible explanation for the blue variable stars.

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